# **Energy-efficient Train Control in Urban Rail Transit: Multi-train Dynamic Cooperation based on Train-to-Train Communication**

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Abstract—With the increasing energy consumption in urban rail transit systems, energy-efficient train operation has been paid significant attention. Considering the application of regenerative braking technology, many studies focus on offline energy-saving train trajectory optimization, in which an accelerating regime is inserted into the train trajectory when there is regenerative braking energy (RBE) that can be utilized. However, in practical operation, train states are dynamic and scheduled trajectory might be useless. This paper proposes a real-time cooperative train control method based on Train-to-Train (T2T) communication technology. According to the train states transmitted through T2T communication, whether there is a train in braking regime is judged. Besides, an accelerating regime is inserted by changing train running modes when there is another train in the braking regime. A cooperative control algorithm is developed to achieve the energy-efficient cooperative control strategy. Besides, simulations based on a real-life metro line demonstrate that the proposed cooperative control method can reduce substation energy consumption and improve the utilization of RBE.

### I. INTRODUCTION

Urban rail transit (URT) is developing rapidly in recent years to meet the increasing passenger demands. However, with the rising energy prices and environmental issues, the huge energy consumption of URT systems have become a grand challenge. It has aroused significant attention to energy conservation of URT systems. Energy-effective train control is an effective method to reduce energy consumption by adjusting train trajectory (or called driving strategy and speed profile) [1].

Energy-efficient train control, as the mainly used method of energy saving in rail transit, has been studied for a long time. Ishikawa [2] uesd the Pontryagain's Maximum Principle (PMP) to lay the first optimal train control theory. Howlett [3] proved mathematically that the energy-efficient train control strategy for a level track with the fixed speed restriction consisted of four regimes: maximum accelerating (MA), coating (CO), cruising (CR), and maximum braking (MB). In addition, Pudney and Howlett [4] introduced the conditions of varying gradients and speed restrictions into the energy-efficient train control problem and got the optimal control strategy based on PMP. Methods in studies [2]-[4] belong to the analytical algorithms, in which the optimal solutions are obtained with complicated computational processes and large computation times. With the development of

computer technologies and train control systems, automatic train operation (ATO) system is widely applied in URT systems. Lots of numerical algorithms are applied to solve the energy-efficient train control problem with shorter computation times, like dynamic programming [5], pseudospectral method and mixed-integer linear programming [6], coast control method [7]. Studies [2]-[7] pay attention to the single train control problem with the objective of reducing the train traction energy consumption.

With the application of regenerative braking technology, the train traction energy consumption can be partly provided by the braking trains with RBE produced. Then, the substation energy consumption can be reduced by improving the utilization of RBE. To improve the utilization of RBE, studies on energy-efficient train control focus on adding accelerating regimes into the train control strategy when there is RBE that can be used. Xun et al. [8] inserted an accelerating regime into the control strategy based on the predicted RBE transfer time. Sun et al. [9], [10] formulated an RBE usage method to adjust the trajectory by partly replacing the original control strategy with CO-MA-CO regime. The method was under the assumption that train braking regimes were predicted based on the field data of the train control system, and the braking information could be obtained in advance. Jin et al. [11] proposed a train trajectory optimization method to improve the utilization of RBE, in which CO-MA regime was inserted into the trajectory according to the pre-calculated substation power flow. Liu et al. [12] discussed the cooperative control for two trains and proposed an energy-efficient control strategy with five regimes (MA, CR/CO, MA, CO, and MB), where an maximum accelerating (MA) regime was inserted to better utilize the RBE. The previous multitrain cooperation methods only optimize the train trajectory before train departure with perfect deterministic information (RBE transfer time [8]-[10], substation power flow [11], train trajectory [12]).

However, the environment of train operation is dynamic and cannot be obtained in advance. A multi-train dynamic cooperation method is critical to achieve the adjustment of the train trajectory when the environment information is updated. Nevertheless, real-time data transmission cannot be realized in the traditional railway control system. In recent years, the technology of enhanced Communication-based Train Control (CBTC) system through T2T communication is applied in URT systems [13]. Different from the traditional CBTC system, the real-time train states, like speed and position, can be exchanged between trains through T2T communication in the novel CBTC system [14].

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The main contribution of this paper is to propose a novel multi-train dynamic cooperation method based on T2T communication for energy-efficient train control URT systems. First, a method to judge whether the train is in the braking regime based on the speed and position states is proposed. This judgment is used to guide the adjustment of the train control strategy. Second, a real-time train control mode adjustment algorithm is built, in which accelerating regimes are inserted dynamically when there is another train in the braking regime. In addition, the advantage of this algorithm is that multiple accelerating regimes can be inserted into the control strategy during the interstation running process for better utilization of RBE.

The rest of this paper is organized as follows. The multitrain cooperation problem based on T2T communication is stated in the next section. The energy-efficient multi-train cooperation method is then proposed. Simulations are then presented to verify the effectiveness of the proposed method. Finally, the conclusion is given.

### II. PROBLEM STATEMENT

### A. Definition of Symbols

For a better understanding of this paper, the main symbols are defined as shown in Table I.

 $\label{eq:table I} \mbox{TABLE I}$  The introduction of the Main Symbols

Symbols	Defination	Measure
		unite
i	Index of train	/
$\mid j \mid$	Index of sattion	/
x	Train running position	m
t	Train running time	s
v	Train running speed	m/s
a	Train accelerationn	m/s <sup>2</sup>
M	Train mass	t
$\rho$	Factor considering the rotating mass	/
$\mid \dot{\eta} \mid$	Efficiency of the train motor	/
$F_t$	Train traction force	kN
$F_b$	Train braking force	kN
$F_{eb}$	Train regenerative braking force	kN
$F_{ab}$	Train air braking force	kN
$R_b$	Basic train running resistance	kN
$R_q$	Line resistance due to track gradient	kN
$\mu$	Utilization rate of regenerative energy	/
T	Operation time	s
$v_m$	Speed restriction	m/s
$E_t$	Train traction energy consumption	kWh
$E_s$	Substation energy consumption	kWh
$P_t$	Power of train traction	kW
$P_a$	Power of train auxiliary system	kW
$P_l$	Power of energy transmission losses	kW
$P_s$	Power of substation	kW
$P_w$	Power of wasted regenerative energy	kW
$P_r$	Power of regenerative energy	kW

### B. CBTC System with T2T Communication

In CBTC systems, trains are usually equipped with ATO systems. The train control strategy is determined in the ATO system based on the static information, like schedule commands, train data and line data. And the control strategy

is generally determined before train departing. For the CBTC system with T2T communication, the train can receive the dynamic information from adjacent trains [15], as shown in Fig. 1. The information transmitted in real-time between trains includes train position and speed, which is mainly used for safety protection. On the other hand, the train control strategy can be optimized in real-time based on the information, then the multi-train dynamic cooperation can be realized.

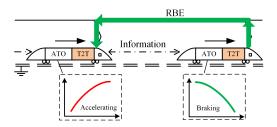


Fig. 1. Multi-train cooperation based on T2T communication

### C. Energy-efficient Multi-Train Dynamic Cooperation

Synchronizing the accelerating and braking regimes is an effective way to improve the utilization of RBE. As shown in Fig 1, the front train is in the braking regime and the latter train is in the accelerating regime, then the RBE from the front train can be absorbed by the latter train. However, if the latter train is not in the accelerating regime, the RBE will be wasted by resistances. An effective way to synchronize the accelerating and braking regimes is multi-train cooperation by adjusting the train control strategy. Inserting an accelerating regime into the control strategy when there is a train in the braking regime is proved to be a useful method to improve the utilization of RBE [8]-[12]. With the help of T2T communication, the periods of braking regimes can be determined by the transmitted train speed and position sates. When the periods of braking regimes are determined, which means there is RBE, which can be utilized, then accelerating regimes can be inserted. Therefore, a method for judging whether the train is in the braking regime and a framework for real-time train control strategy adjustment are both necessary to achieve the multitrain dynamic cooperation.

## III. ENERGY-EFFICIENT MULTI-TRAIN DYNAMIC COOPERATION ALGORITHM BASED ON T2T COMMUNICATION

### A. Energy-efficient Control for Single Train

The optimization of energy-effective control for multi-train is based on the one for single train, which should be worked out by building the optimization model and proposing the solution method. The optimization model of energy-effective control for single train running in a certain interstation  $[x_{sta}, x_{end}]$  with a pre-given time interval  $[t_{sta}, t_{end}]$ , can

be expressed as follows:

$$\begin{cases} \min & E_{t} = \frac{1}{3600} \int_{t_{sta}}^{t_{end}} P_{t}(t)dt \\ \text{s.t.} & v(x) < v_{m}(x) \\ & v(x_{sta}) = v(x_{end}) = 0 \\ & t(x_{sta}) = t_{sta} \\ & t(x_{end}) = t_{end} \\ & 0 \le F_{t} \le F_{t,max} \\ & 0 \le F_{b} \le F_{b,max} \end{cases}$$
(1)

where,  $P_t(t)$  can be calculated as:

$$P_t(t) = F_t(v(t))v(t)/\eta \tag{2}$$

For the single train operation,  $x_{sta}$  and  $x_{end}$  are the position of the departure and arrival stations respectively,  $t_{sta}$  and  $t_{end}$  are the departure and arrival times respectively. And the motion of the train can be described by the following continuous-space model:

$$\begin{cases} \frac{dv}{dx} = \frac{F_t - F_b - R_b(v) - R_g(x)}{M\rho v} \\ \frac{dt}{dx} = \frac{1}{v} \end{cases}$$
 (3)

Additionally,

$$R_b(v) = b_1 + b_2 v + b_3 v^2 (4)$$

$$R_g(x) = Mg\sin(\theta(x)) \tag{5}$$

where,  $b_1$ ,  $b_2$  and  $b_3$  are all non-negative constants, depending on the train characteristics. g is gravitational acceleration,  $\theta(x)$  is the slope of gradient. The train braking force  $F_b$  consists of two parts:

$$F_b(v) = \begin{cases} F_{ab} & v \le v_{eb} \\ F_{eb} & v > v_{eb} \end{cases} \tag{6}$$

There is a special speed value  $v_{eb}$ , when the train speed is larger than it, the train braking force is provided by the regenerative braking force, otherwise, the regenerative braking force is equal to zero and the train braking force is provided by the air braking force.

With the steep gradient and the significant speed restriction ignored, the energy-efficient train control strategy for single train can be obtained based on PMP, consisting of MA, CR, CO, and MB regimes [4], as shown in Fig. 2. In the literature [11], we have proposed a coast control method to calculate the energy-efficient train control strategy with four regimes. To summarize, the coast control method includes two parts: the calculating of MaxPow trajectory (TMP) and the calculating of trajectory with CO regime (TCO). In the calculating of TMP, the train control strategy includes three regimes: MA, CR and MB, to get close to the speed restrictions as soon as possible. Based on the TMP, CO regime is inserted to reduce the train traction energy consumption. Meanwhile, the starting point of CO regime is searched to meet the constraint of the pre-given runtime.

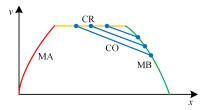


Fig. 2. Energy-effective train control strategy for single train

#### B. Optimization Model for Multi-train Cooperation

For the energy-saving train control problem for multi-train, the substation energy consumption is used to measure the total energy consumption [16]. Then, the objective function of multi-train cooperation can be described as:

$$\min E_s = \frac{1}{3600} \int_{t_{sta}}^{t_{end}} P_s(t) dt$$
 (7)

where,  $P_s$  can be calculated as:

$$P_s = \sum P_t - \left(\sum P_r - \sum P_w\right) + \sum P_l + \sum P_a \quad (8)$$

and  $P_r$  can be calculated as:

$$\min P_r = \eta F_{eb} v \tag{9}$$

Specially,  $P_l$  is simplified to be zero,  $P_a$  is simplified to be constant,  $P_w$  can be calculated as:

$$P_w = \begin{cases} 0 & \sum P_t + \sum P_a + \sum P_l > \sum P_b \\ \sum P_b - \sum P_t - \sum P_a - \sum P_l & \text{else} \end{cases}$$
 (10)

which means when the RBE is not fully used by accelerating trains, it will be wasted.

Meanwhile, the control of each train should meet the constraints in Eq. 1. In addition, the safety interval needs to be maintained between successive trains to avoid conflicts, which can be described as the following constraint:

$$A_{i+1,i} - D_{i,i} > T_{safe}$$
 (11)

where  $D_{i,j}$  is the departure time of train i at station j and  $A_{i+1,j}$  is the arrival time of train i+1 at station j,  $T_{safe}$  is the safe time interval between successive trains.

According to Eq. 8, energy consumption can be reduced by minimizing the wasted RBE. In other words, improving the utilization of RBE can achieve the energy-saving effect of multi-train control.

### C. Judgment of Train Braking Regime based on T2T Communication

With the help of T2T communication, trains can obtain the position and speed data from the adjacent trains. However, it is unable to use these raw data to guide the train control. For the energy-efficient multi-train cooperation, we pay attention to whether a train is in the braking regime. When another train is in the baking regime, the train control strategy should be adjusted by adding accelerating regime to absorb RBE. Therefore, a method to judge whether the train is in the braking regime based on the position and speed states is proposed, as shown in Fig. 3. First, the train position x is used to

judge whether the train is in the braking range  $[x_{b,j}, x_{end,j}]$ , where  $x_{b,j}$  and  $x_{end,j}$  are the starting and ending points of the MB regime in the intersection j respectively. Second, the train speed v is used to judge whether the train is in the braking regime. The train acceleration a is calculated based on the speed as:

$$a = \frac{v - v'}{\Delta t} \tag{12}$$

where v' is the speed of the previous state,  $\Delta t$  is the state update time interval. When the train acceleration a is smaller than the pre-set acceleration  $a_b$ , the train can be judged to be in the braking regime. Third, the speed v should be larger than the speed value  $v_{eb}$ , otherwise, there is no RBE can be utilized, as shown in Eq. 6. Thus, only if the above three conditions are all met, there is RBE can be utilized.

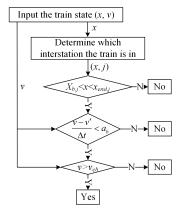


Fig. 3. Flowchart for judging whether train is in the braking regime

### D. Multi-train Dynamic Cooperation Algorithm

In this algorithm, we divide the train running processes into three control modes:

- TMP: MaxPow trajectory.
- TCO: Trajectory with CO regime.
- STOP: Stopping at a station.

and the operational messages are divided into four types:

- ATS-dep: The train departure command from the Automatic Train Supervision (ATS) system.
- RBE-yes: The message means that there are other trains in braking regimes and RBE can be utilized.
- RBE-no: The message means that there is no train in the braking regime and no RBE can be utilized.
- Train-stop: The train stopping status from the trackside equipment.

Based on the operational messages, the principle of train control mode conversion is proposed, as shown in Fig. 4. When the "STOP" train receives the message "ATS-dep", then it departures from the station with the control mode "TCO". The train keeps the "TCO" control mode when the operational message is "RBE-no". On the other hand, the train switches to "TMP" control mode when the operational message changes to "RBE-yes", and the accelerating regime is taken to absorb the RBE, as shown in Fig. 5. When the

operational message turns into "RBE-no", the train changes to "TCO" mode again. Thus, according to the state of other trains, whether there is a train in the braking regime is judged, and the train control mode switches between "TCO" and "TCM" in the interstation running process. Then, multiple accelerating regimes can be inserted to improve the utilization of RBE. When the message "Train stop" appears, the train changes to the "STOP" mode and waits for the message "ATS-dep".

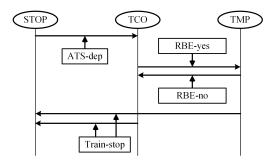


Fig. 4. Train control mode conversion principle based on the operational messages

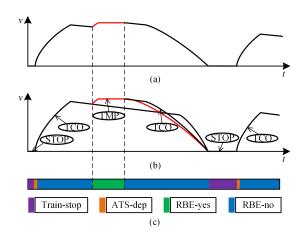


Fig. 5. Energy-efficient control strategy for multi-train dynamic cooperation. (a) Optimal train trajectory. (b) Train control modes conversion. (c) Operational messages

The conclusion of the proposed multi-train dynamic cooperation algorithm based on T2T communication is shown in Fig. 6. The train control modes are switched based on the operational messages, and the train speed is updated to obtain energy-saving. Different with the single train control strategy, as shown in Fig. 2, the control strategy of multi-train dynamic cooperation is changing in real-time, as shown in Fig. 5. When the train control mode changes, TMP or TCO needs to be recalculated.

### IV. SIMULATIONS

### A. Simulation Parameters

Simulations are based on the data of one of the Guangzhou Subway Lines. The train running in the subway line is an A-type EMU with a mass of 339.6t, the factor considering

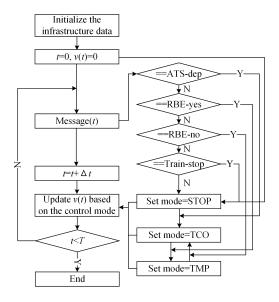


Fig. 6. Flowchart for multi-train dynamic cooperation algorithm. Note that the algorithm is executed in the ATO system, and the message represents the dynamic information from outside the ATO system

the rotating mass is 0.057, the efficiency of train motor is 0.85, the auxiliary power is 200kW. The traction force, braking force, and basic resistance are described as Eq. 13-15 respectively. The safe time interval between successive trains is set to 20s.

$$F_{t,max}(v) = \begin{cases} 400 & 0 \le v \le 11.1\\ 4440/v & 11.1 < v \le 15.3\\ 67932/v^2 & 15.3 < v \le 22.2 \end{cases}$$

$$F_{eb,max}(v) = \begin{cases} 0 & 0 \le v \le 1.4\\ 389 & 1.4 < v \le 22.2 \end{cases}$$
(13)

$$F_{eb,max}(v) = \begin{cases} 0 & 0 \le v \le 1.4\\ 389 & 1.4 < v \le 22.2 \end{cases}$$
 (14)

$$r(v) = 9.067 + 0.01729v^2 (15)$$

In the simulations, the running processes of four trains (train 1, 2, 3, and 4) between three stations (station 1, 2, and 3) are analyzed. For the T2T communication, trains send running states to the successive trains, e.g. train 2 sends running states to train 1 (the front train) and train 3 (the latter train). In addition, The speed restriction and the change of gradient are shown in Fig. 7. The positions of station 1, 2, and 3 are 0m, 1631m, 2569m respectively, the runtime from station 1 to station 2 and from station 2 to station 3 are 110s and 75s respectively, the headways of train 2, train 3, train 4 are 150s, 135s, 120s respectively, and the dwell time at station 2 is 30s. Simulations are tested under the MATLAB environment on a personal computer with Intel Core i5 2.30 GHz CPU and 8GB RAM.

### B. Simulation of Multi-train Dynamic Cooperation Method **Testing**

A simulation based on a train running process between station 1 and station 2 with pre-set operational messages is built, aiming to test the proposed multi-train dynamic cooperation method. The pre-set operational messages are as shown in Fig. 8. There are two "RBE-yes" messages

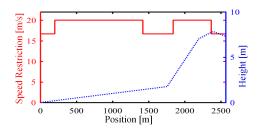


Fig. 7. Speed restriction and change of gradient

in the time interval [40, 50]s and [70, 75]s respectively, in which two accelerating regimes are inserted into the control strategy of multi-train cooperation as shown in Fig. 8. In the first time interval, the train reaches the speed restriction, thus "CR" regime is taken to partly absorb RBE and keep safety. Meanwhile, in the second time interval, "MA" regime is first taken to better utilize RBE before the train reaches the speed restriction, and then "CA" regime is taken. Therefore, a safe train trajectory with multiple suitable accelerating regimes can be calculated by applying the proposed multitrain dynamic cooperation method.

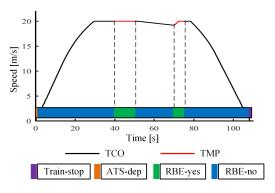


Fig. 8. Train trajectories of different control methods among two stations

### C. Simulation of Multiple Trains Running in the Same Substation

In this section, the cooperative control method is compared with the independent control method. Trajectories of those two methods are shown in Fig. 9. The cooperative control means that the train trajectory is optimized based on the states of itself and other trains, which is realized by the proposed multi-train dynamic cooperation method. The independent control means that the train trajectory is optimized only based on the states of itself, which is realized by the coast control strategy with four regimes proposed in the literature [11]. As shown in Fig. 9, accelerating regimes are inserted in trajectories of cooperative control, like the trajectory of train 3 in the time interval [300, 400]s. Due to the insertion of accelerating regimes, the substation energy consumption of the cooperative control is reduced by 0.7% compared with the independent control, and the utilization rate of RBE is improved 6.6%, as shown in Table II. In addition, comparing the cooperative control with the independent control, the total train operation energy consumption

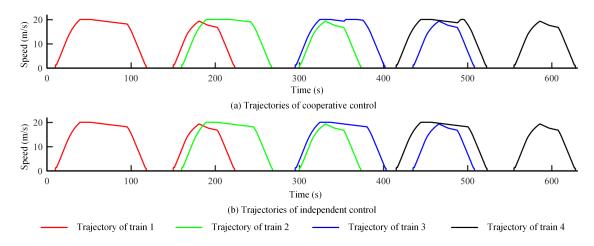


Fig. 9. Trajectories of cooperative control and independent control

has increased. It indicates that the control strategy of the independent control is more energy-efficient than the control strategy of the cooperative control in terms of single-train trajectory optimization. However, considering the utilization of RBE, the cooperative control method is more energy-efficient in terms of multi-train trajectory optimization. Considering the computation time, the average computation times of independent control and cooperative control are both kept at a small value, which are meet the real-time computation requirements of trajectory optimization.

TABLE II
ENERGY CONSUMPTION OF COOPERATIVE AND INDEPENDENT CONTROL

Items	Cooperative control	Independent control
Substation energy consumption [kWh]	282.0	284.1
Utilization rate of RBE [%]	29.7	23.1
Total train operation energy consumption [kWh]	315.8	308.9
Average computation time [s]	1.20	1.96

### V. CONCLUSIONS

In this paper, we proposed an energy-efficient cooperative train control method based on T2T communication for URT systems. The train trajectory is optimized by inserting accelerating regimes according to the state of other trains. In addition, the trajectory can be optimized in real-time and multiple accelerating regimes can be inserted. The simulation results show that the substation energy consumption can be reduced, and the utilization of RBE can be improved by applying the proposed control method. In future work, a more accurate train trajectory optimization method should be proposed considering the power of accelerating/braking regimes.

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